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Localization and Delocalization in Periodic One-Dimensional Dynamic Systems

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Localization and Delocalization in Periodic One-Dimensional Dynamic Systems

by
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Localization and Delocalization in Periodic
One-Dimensional Dynamic Systems

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Abstract

The impulse response function of ribbed membrane-like panels is derived. A model of such a panel in which certain interactions among the ribs are removed is constructed. The removed interactions are those that allow a rib to identify the dispositions of the other ribs. It is argued that the impulse response function describing this model just fails, by definition, to account for the phenomena associated with pass and stop bands. The characteristics of this model are then used to explain the phenomenon of localizations that occurs, at frequencies that lie within the pass bands, when the periodicity of the ribs is disturbed. Similarly, this model is used to explain the phenomenon of delocalizations that occurs, at frequencies that lie within the stop bands, when the periodicity of the ribs is disturbed. The results of computer experiments that exhibit these types of localizations and delocalizations are cited.

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I Introduction

The response $\tilde{\tilde{v}}(\underset{\sim}{x},t)$ of a dynamic system may be related to the external drive $\tilde{\tilde{p}}_e(\underset{\sim}{x},t)$ that excites it, in terms of an impulse response function $\tilde{\tilde{g}}(\underset{\sim}{x}\,|\,\underset{\sim}{x}', t\,|\,t')$; namely,

$$\tilde{\tilde{v}}(\underset{\sim}{x},t) = \int \tilde{\tilde{g}}(\underset{\sim}{x}\,|\,\underset{\sim}{x}', t\,|\,t')\, d\underset{\sim}{x}'\, dt'\, \tilde{\tilde{p}}_e(\underset{\sim}{x},t) \quad , \tag{1a}$$

where t is the temporal variable and $\underset{\sim}{x}$ is the vector variable in the spatial domain that spans the dynamic system; e.g., if the spatial domain spans a surface then $\underset{\sim}{x} = \{x, y\}$ and $d\underset{\sim}{x}' = dx'\, dy'$. Equation (1a) is pure if the impulse response function is pure; the impulse response function is pure if it is dependent solely on quantities and parameters of the dynamic system and is completely independent of the response and the external drive. Equation (1a) may be advantageously and conveniently expressed in domains that are either partial or full transforms of the $\{\underset{\sim}{x}, t\}$ - space. For example, employing Fourier transformations on equation (1a) one may formally state

$$\tilde{v}(\underset{\sim}{x}, \omega) = \int \tilde{g}(\underset{\sim}{x}\,|\,\underset{\sim}{x}', \omega\,|\,\omega')\, d\underset{\sim}{x}'\, d\omega'\, \tilde{p}_e(\underset{\sim}{x}',\omega') \quad , \tag{1b}$$

$$v(x, \underset{\sim}{\omega}_2) = \int g(x\,|\,x', \underset{\sim}{\omega}_2\,|\,\underset{\sim}{\omega}'_2)\, dx'\, d\underset{\sim}{\omega}'_2\, p_e(x',\underset{\sim}{\omega}'_2) \quad , \tag{1c}$$

$$V(k, \underset{\sim}{\omega}_2) = \int G(k\,|\,k', \underset{\sim}{\omega}_2\,|\,\underset{\sim}{\omega}'_2)\, dk'\, d\underset{\sim}{\omega}'_2\, P_e(k',\underset{\sim}{\omega}'_2) \quad , \tag{1d}$$

where typically

$$v(x, \underset{\sim}{\omega}_2) = (2\pi)^{-1} \int \tilde{\tilde{v}}(\underset{\sim}{x}, t)\, dy\, dt\, \exp\left[i\,(y k_y - \omega t)\right] \quad , \tag{2a}$$

$$g(x\,|\,x',\underset{\sim}{\omega}_2\,|\,\underset{\sim}{\omega}'_2) = (2\pi)^{-2} \int \tilde{\tilde{g}}(\underset{\sim}{x}\,|\,\underset{\sim}{x}', t\,|\,t')\, dy\, dt\, dy'\, dt'\, \exp\left[i\,(y k_y - y' k'_y - \omega t + \omega' t')\right] \quad , \tag{2b}$$

the wave vector variable $\underline{k} = \{k, k_y\}$ is the Fourier conjugate of the spatial vector variable $\underline{x} = \{x, y\}$, the frequency variable ω is the Fourier conjugate of the temporal variable t, and $\underline{\omega}_2 = \{k_y, \omega\}$. Situations arise in which the impulse response function is stationary in some or all the dependent variables. The usefulness of the transform procedure in those variables becomes immediately apparent. For example, if the impulse response function $\bar{\bar{g}}(\underline{x} \mid \underline{x}', t \mid t')$ is stationary with respect to y and t, so that

$$\bar{\bar{g}}(\underline{x} \mid \underline{x}', t \mid t') \rightarrow (2\pi)^{-1} \bar{\bar{g}}(x \mid x', y - y', t - t') \ , \tag{3a}$$

then

$$g(x \mid x', \underline{\omega}_2 \mid \underline{\omega}'_2) \rightarrow g(x \mid x', \underline{\omega}_2) \ \delta(k_y - k'_y) \ \delta(\omega - \omega') \ , \tag{3b}$$

and equation (1c) and (1d) reduce to

$$v(x, \underline{\omega}_2) = \int g(x \mid x', \underline{\omega}_2) \, dx' \ p_e(x', \underline{\omega}'_2) \ , \tag{4a}$$

$$V(k, \underline{\omega}_2) = \int G(k \mid k', \underline{\omega}_2) \ dk' \ P_e(k', \underline{\omega}'_2) \ , \tag{4b}$$

respectively. Were the impulse response function to be stationary also with respect to x, so that

$$g(x \mid x', \underline{\omega}_2) \rightarrow (2\pi)^{-1} g_\infty(x - x', \underline{\omega}_2) \ , \tag{5a}$$

then

$$G(k \mid k', \underline{\omega}_2) \rightarrow G_\infty(k, \underline{\omega}_2) \ \delta(k - k') \ , \tag{5b}$$

and equation (1d) reduces to

$$V_\infty(k, \underline{\omega}_2) = G_\infty(k, \underline{\omega}_2) \ P_e(k, \underline{\omega}_2) \ , \tag{6}$$

where the infinity (∞), as a subscript, indicates that the quantity is stationary in the "real space" in which the dynamic system is described; e.g., in equation (6), with respect to $\{x, y, t\}$. The simplicity of equation (6) is obvious. If the impulse response function that

relates the response to the external drive is fully stationary then in the transformed space the relationship becomes algebraic; e.g., equation (1a) versus equation (6).[1] In spite of this simplicity there are several phenomena that can be accounted for by models of dynamic systems that admit to fully stationary impulse response functions. In Reference 1, which deals with the impulse response function of panels, it is shown that the transfer function on, and the specular reflection and transmission coefficients of uniform panels can be readily derived off equations such as equation (6) [1,2].[2] In spite of the welcomed simplicity, it is recognized that there are many phenomena of interest that cannot be accounted for by models of dynamic systems that admit to fully stationary impulse response functions. These phenomena can be accounted for only if specific deviations and/or variations from uniformity are incorporated [3]. Nonuniformities result in impulse response functions that are not stationary in some or all the dependent variables. Nonuniformities of these kind may, for instance, explain and account for the complex transfer functions on ribbed panels, and their nonspecular reflection and transmission functions [1,2]. Of course, to explain and account for some of the phenomena, more and more elaborate and compounded models may have to be constructed. The impulse response functions describing such models may not only be nonstationary in more and more variables, but their very functional forms may be horrendous. Often in such situations one may effectively rely on statistical techniques and measures to reduce and mollify the descriptions. It is, however, recognized that statistical procedures must be applied with caution; the simplicity afforded may be blinding. This is particularly true when the introduced nonuniformities are identical and periodic in some or all of the dependent variables, or nearly identical and/or periodic [3]. [Nonuniformities that are identical are clones.] It may then behoove one to construct models that may simulate, in part, the statistical models so that one may be able to authenticate the behavior that the statistical models suggest. In this paper the task set is rather modest. The nonuniformities are introduced on bare, uniform, and membrane-like panels by the attachment of ribs.

A rib introduces a one-dimensional spatially localized nonuniformity that lies along the y-direction; see Figure 1. The response behavior of such ribbed panels is investigated to illustrate the phenomena of localizations and delocalizations. The results of statistical techniques and procedures applied to these models are also cited and discussed. The purpose is not to comprehensively treat a large and significant area of the behavior of dynamic systems, but rather, to discuss a point of view that may be new.

The class of pure impulse response functions that describe nonuniformities introduced on a uniform membrane-like panel by a one-dimensional array of ribs may be stated in the form

$$g(x \mid x') = g_\infty(x \mid x') - g_s(x \mid x') = g_\infty(x \mid x') \, [1 - s\,(x \mid x')] \;;$$
$$g_s(x \mid x') = g_\infty(x \mid x') \, s\,(x \mid x') \;;$$
$$g_s(x \mid x') = \sum_n \sum_r g_\infty(x \mid x_n) \, T_{nr} \, g_\infty\,(x_r \mid x') \quad , \tag{7}$$

where the suppression of the dependence of quantities and parameters on $\underline{\omega}_2$, k_y, or ω, in this and subsequent equations, is a matter of abbreviation; e.g., $g\,(x \mid x', \underline{\omega}_2) = g\,(x \mid x')$ and $T_{nr}\,(\underline{\omega}_2) = T_{nr}$; $g_\infty\,(x \mid x')$ is the line transfer admittance, on a uniform panel, from the line drive position x' to the (line) observation position x, $g_\infty\,(x \mid x) = g_\infty\,(x' \mid x')$ is the line admittance of the uniform panel, x_n is the (line) position of the (n)th rib, and T_{nr} is the coupling line impedance between the (r)th and the (n)th ribs; see Figure 1 and Reference 1. In the absence of ribs, $g_s\,(x|x')$ vanishes, and the impulse response function appropriately become stationary also with respect to x. One is reminded that the impulse response function $g\,(x|x')$ is already stationary with respect to y and t. It is, therefore, apparent that $g_s\,(x|x')$ exclusively and completely accounts for the presence of the ribs. Thus, the impulse response function of a ribbed panel can be expressed in terms of the superposition of the impulse response functions of the uniform panel and that generated by the ribs. The casting of equations (7) in the form of $g_s\,(x|x')$ or $s\,(x|x')$ emphasizes that the influence of

the ribs can be cast either in the form of terms or of factors. Although the two forms are simply related, there are, nonetheless, situations in which one form may exhibit interpretive advantages over the other. Indeed, the superposition argument just advanced is one example of such situations. Another example relates to the aliasing in the wavenumber k of the Fourier transform of s (x|x′). When equation (7) is cast in the entire spectral space, the $\{k, (\underline{\omega}_2\}$-space, it is found that S (k|k′) [the Fourier transform of s (x|x′)] and not G_s (k|k′) [the Fourier transform of g_s (x|x′)] is aliased with respect to the harmonics of the fundamental wavenumber, κ_1, of the periodic separations between adjacent ribs; e.g., $j\kappa_1$ is the (j)th harmonic. The aliasing of S (k|k′) in k is defined by $S(k + \kappa_j|k') = S(k|k')$. The aliasing phenomenon is discussed in some detail in Reference 4.

The quantities and parameters that appear in equation (7) may be more explicitly stated:

$$T_{nr} = [R_n / g_\infty (x_n \mid x_n)]\, C_{nr}; \qquad \underline{\underline{C}} = (C_{nr}) = (\underline{\underline{B}})^{-1} \quad , \tag{8}$$

$$\underline{\underline{B}} = \big(B_{ji}\big) = \big(\delta_{ji} + a_\infty (x_j \mid x_i)\, R_i\, (1 - \delta_{ji})\big) \; ;$$
$$a_\infty (x \mid x') = [g_\infty (x \mid x') / g_\infty (x \mid x)] \quad , \tag{9}$$

$$R_n = Z_n\, g_\infty (x_n \mid x_n)\; [1 + Z_n\, g_\infty (x_n \mid x_n)]^{-1} \quad , \tag{10}$$

$$g_\infty (x \mid x') = (2\pi)^{-1} \int G_\infty (k)\, dk \exp[-ik(x - x')] \quad , \tag{11}$$

where $G_\infty(k)$ is the admittance of the uniform panel in the $\{k, \underline{\omega}_2\}$-space, and Z_n is the line impedance that the (n)th rib presents to the panel [1]. The purity of the impulse response function g(x | x′) of the ribbed panel, as stated in equation (7), is thus made explicit.

Recently the authors derived an impulse response function for a ribbed panel that is closely akin to that stated in equations (7) through (11) [1]. In this derivation the impulse

response function of a panel can accommodate line moment impedances as well as environmental loadings. The impulse response function of a fluid loaded plate responding in flexure can thus also be described. Equations (7) through (11) may then be extended to accommodate this more elaborate form of the impulse response function of the panel. Further, one may quite readily express equations (7) through (11) in the abstract "state-operators" form and in the notations adopted by Sornette in a recent series of review papers dealing with the "acoustic waves in random media" [5-7]. Indeed, these papers bear directly and substantially on the present paper; e.g., the recognition that a marked simplification may be obtained if the analysis is limited to one spatial dimensionality; the analysis stated in equations (7) through (11) becomes indeed one-dimensional if the wavenumber k_y is fixed; e.g., fixed at zero. The one dimensionality is not only simpler but it also accentuates the presence of the very phenomena that is of interest here [5-7]. For the purposes of the paper, it is not essential to bring to bear either the greater elaborations afforded by Reference 1; e.g., permitting the generation of moments and environmental loadings, or the more elegant mantle for the formalism employed by Sornette; equations (7) through (11) remain as are.

II R– and NR– models

It may be useful to discuss and highlight a few of the notions and ideas that formulate the phenomena of interest in this paper prior to indulging in details and computational displays. However, for those who prefer to see computational displays as they proceed, references to the appropriate figures are made. It is also to be noted that although the formalism expressed and further developed in this paper is capable of handling an extensive range of parametric values, in this and the succeeding sections, the statements, arguments, and illustrations are largely directed toward the moderate range of values for the parameters that describe the panel and the ribs. Therefore, for the most part the reflection coefficient of a rib is near unity, the number of ribs is one or two scores, and the loss factor of the panel is in the vicinity of 10^{-2}. (Considerations of the more extreme parametric values will be discussed under a separate cover [5, 6].) Again, hereafter the basic equations are those expressed in equations (7) through (11). These equations describe the impulse response function of the basic model comprising a ribbed membrane-like panel [1]. This model is designated the R– model. It is advantageous and instructive to artificially contrive a model in which "return" interactions among the ribs are excluded; only "nonreturn" interactions are allowed. This model is designated NR– model. Nonreturn interactions are defined here as those interactions that do not disclose to a rib information concerning the dispositions of the others. That is, a wave that interacts with that specific rib is not allowed to propagate to another rib and <u>return</u> to that initial rib; see Figure 2. All other forms of interactions among the ribs are included.[3] An examination of equations (7) through (11) and Figure 2 reveals how the NR– model may be constructed. It is defined that x' lies between the adjacent ribs at x_R and x_{R+1}; $x_R < x' < x_{R+1}$, and that x lies between the adjacent ribs at x_N and x_{N+1}; $x_N < x < x_{N+1}$, see Figure 1. Now, to ensure the exclusion of return interactions and to retain all others, one requires that the elements of the coupling

impedance matrix $\underset{\approx}{T}$ in equations (7) through (11) be replaced by the corresponding elements of the matrix $\underset{\approx}{T}^0$ which may be conveniently defined in the form

$$\underset{\approx}{T}^0 = \left(T^0_{nr}\right) \;;\quad T^0_{nr} = [R_n/g_\infty(x_n \mid x_n)]\, C^0_{nr} \quad ;$$

$$\underset{\approx}{C}^0 = \left(C^0_{nr}\right) \;;\quad \underset{\approx}{C}^0 = (\underset{\approx}{B}^0)^{-1} \quad ;$$

$$\underset{\approx}{B}^0 = \Big(\delta_{ji} + B_{ji}\ [U\,(j-i)\,U\,(j-R)\,U\,(N+1-j)\,U\,(x-x')$$

$$+ U\,(i-j)\,U\,(i-N)\,U\,(R+1-i)\,U\,(x'-x)]\ (1-\delta_{ji})\Big) \qquad , \tag{12}$$

where U is the unit step function. [cf. equations (8) and (9).] For example, if $x' < x$ the matrix $\underset{\approx}{B}^0$ is of the form

$$\underset{\approx}{B}^0 = \left(\begin{array}{c:c:c} \begin{matrix}\delta_{11} & \underset{\approx}{O} \\ \underset{\approx}{O} & \ddots \end{matrix} & \underset{\approx}{O} & \underset{\approx}{O} \\ \hdashline & \begin{matrix}\delta_{RR} & & \\ & \ddots & \underset{\approx}{O} \\ & & \ddots \end{matrix} & \underset{\approx}{O} \\ \hdashline & & \begin{matrix}\delta_{N+1\ N+1} & & \underset{\approx}{O} \\ & \ddots & \\ \underset{\approx}{O} & & \ddots \quad \delta_{FF}\end{matrix} \end{array}\right) , \tag{13}$$

where F is the total number of ribs in the array, $\underset{\approx}{O}$ designates a submatrix with zero elements, and a "blank" submatrix is filled with the elements B_{ji} as prescribed in equation (12). For $x < x'$, $\underset{\approx}{B}^0$ is the mirror image of equations (13), the mirror is placed along the diagonal of the matrix. The $\underset{\approx}{T}^0$ is of similar construction. In particular, the zero submatrices occupy the same elemental positions. This property of $\underset{\approx}{T}^0$ is essential in the

structure of the NR– model. It is clear that the contrived NR– model renders the interactions among the ribs rectified, e.g., $T^0_{nr} \neq 0$ but $T^0_{rn} = 0$; an artificial construction indeed. The NR– model is useful in that the impulse response function that describes it is just devoid of phenomena that feed on return interactions among the ribs. Again, return interactions in the context of this paper refer to the information that a rib may acquire, by interactions, concerning the dispositions of the other ribs. It is noted that a single mode of propagation is assigned to the panel and that environmental loadings are neglected. Therefore, such information can be communicated here solely by this single mode of propagation.[4] It is common knowledge that the pass and stop bands phenomena in ribbed panels are associated with coherent interactions among the ribs [8, 9]. Such interactions need be, by definition, return interactions. Therefore, these phenomena are expected to be absent in the NR– model. [cf. Figures 3a and b versus Figures 4a and b, respectively.] Moreover, the pass and stop bands phenomena are exhibited when the separations between adjacent ribs are maintained equal so that the phases in the return interactions can be, in certain frequency bands, coherently superposed to cause reinforcements or cancellations in the response of the ribbed panel.[5] Arguments of this kind can be further employed to decipher the mechanism that causes the pass and stop bands to fade when, for example, the positions of the ribs are deviated and/or varied so that the separations between adjacent ribs are no longer equal [3]. [cf. Figure 3b versus Figure 5a.] When the strict periodicity of the ribs is violated the phase information acquired by a rib of the dispositions of the others may no longer be made coherent and the interactions when superposed may not cause either substantial reinforcement or substantial cancellations. The phase mixing due to these deviations and/or variations may, therefore, cause the pass and stop band to fade [3, 10]. Considerable phase mixing often leads to descriptions that are commensurate with descriptions of models in which the information of phases is apriori suppressed or removed. Indeed, phase mixing tends to render return interactions as ineffective as the nonreturn interactions. One then expects that when the pass and stop bands fade in the

manner just discussed, the R– model may yield an impulse response function that is more commensurate with that yielded by the NR– model.[6] [cf. Figure 5a versus Figure 4b.] One may further surmise in this vein that when the positions of the ribs are statistically distributed, the resulting impulse response function of the R– model will resemble that of the NR– model.[7] If the number of ribs is high enough and the damping is low enough, this resemblence should emerge even in a single random selection; a single member of the ensemble.[5] [cf. Figure 5a versus Figure 5b.] Notwithstanding that the impulse response functions of the NR– model, and of the statistical R– model are rather insensitive to modest, and to further random disturbances in the positions of the ribs, respectively. [cf. Figure 5b versus Figure 4b.] Finally, the influence of damping in the uniform panel, on the relationships between the R– and the NR– models is briefly considered. Since damping subdues interactions between ribs, especially those that are separated farther apart from each other, and since return interactions are, by definition, of longer range than the corresponding nonreturn interactions, it follows that increase in damping will subdue more and more the pass and stop bands phenomena. [cf. Figure 2.] It is then expected that the impulse response function of an R– model will approach more and more that of a corresponding NR– model as damping is increased. [cf. Figures 3b and 3c versus Figure 4b.] Thus, if one is interested in investigating or encouraging phenomena that are associated with pass and stop bands, in addition to ensuring that the periodicity of the ribs is strictly obeyed, that the ribs possess high enough reflection coefficients, and that their number is high enough, light damping is essential [5-9].

Observation has it that if x' is chosen within the aperture of an array of periodic nonuniformities, the magnitude of the impulse response function $g(x|x', \omega_p)$ of the dynamic system is macroscopically flat as a function of the normalized separation $[(x-x')/b]$ when the frequency ω is chosen to be in a pass band, where $\omega = \omega_p$ [5-9]. [cf. Figure 6a.] The macroscopic unit scale exceeds the nominal separation b between adjacent ribs. It is further observed that when random deviations are imposed on

the periodicity, a change occurs in the response behavior. In particular, the response now drops as $[\,|x-x'|\,/\,b\,]$ is increased, and an apex is formed at $[\,(x-x')\,/\,b\,] \simeq 0$ [cf. Figure 6a versus Figure 6b.] This phenomenon is related to Anderson localizations [5-7, 11]. [cf. Figure 6c.] It is observed that if these deviations are significant enough, the behavior of the resulting impulse response function becomes commensurate with the corresponding impulse response function of the NR– model. [cf. Figure 6b.] In this sense Anderson localizations may be viewed to arise because the random deviations imposed on the periodicity, impose phase mixing of the kind discussed earlier. Moreover, it appears that the magnitude of the impulse response function $g(x|x', \omega_s)$ of a dynamic system that possesses periodic nonuniformities, is macroscopically highly peaked as a function of the normalized separation $[\,(x-x')\,/\,b\,]$ when the frequency ω is chosen to be in a stop band, where $\omega = \omega_s$. The apex of the peak is at $[\,(x-x')\,/\,b\,] \simeq 0$. [cf. Figure 6a.] It is further seen that when random deviations are imposed on the periodicity, a change occurs in this response behavior. In particular, the response now flattens out, with the peak still holding at $[\,(x-x')\,/\,b\,] \simeq 0$. [cf. Figure 6a versus Figure 6b.] This phenomena may be dubbed delocalization.[8] [cf. Figure 6c.] It is observed that if these deviations are significant enough, the behavior of the resulting impulse response function becomes, again, commensurate with the corresponding impulse response function of the NR– model. [cf. Figure 6b.] In this sense this delocalization can be viewed to arise because the random deviations imposed on the periodicity, impose, again, phase mixing. Continuing in this vein, it is noted that if one starts with a periodic NR– model as the foundation, the introduction of the naturally occurring return interactions will cause delocalizations (recovered localizations) at the frequencies ω_p that lie in the pass bands and localizations (recovered delocalizations) at the frequencies ω_s that lie in the stop bands. [cf. Figures 6a and b.] In this format one may claim that an Anderson localization is the recovery of a delocalization.

III Computer Experiments

This section provides a few examples designed to support some of the statements and arguments presented in the preceding sections. For this purpose the uniform membrane-like panel is defined in terms of the surface impedance

$$G_\infty(k) = \{i\omega m\,[Y^2 - (k/k_p)^2]\}^{-1} \;;$$
$$Y^2 = [1 - (k_y/k_{py})^2] \quad , \tag{14}$$

where m is the mass per unit area and $\underset{\sim}{k}_p = \{k_p, k_{py}\}$ is the free wave vector on the panel. The free wavenumber k_p may, in this paper, assume one of two simple forms

$$k_p = k_{po}(1 - i\eta_p) \;\; ; \;\; k_{po}^2 = \begin{cases} (\omega/c_\ell)^2 \quad , & (15a) \\ (\omega\omega_c/c^2) \quad , & (15b) \end{cases}$$

where c and c_ℓ are designated speeds, η_p is the loss factor in the panel, and ω_c is a frequency scale factor. Equation (15a) describes a panel that is membrane-like; e.g., a plate responding longitudinally. On the other hand, equation (15b) describes a membrane that simulates a plate responding in flexure; in this description ω_c may be equated to the critical frequency of the plate with respect to c, where c may be considered the speed of sound in a fluid of negligible density [12]. This condition on the fluid is necessary in order to satisfy the assumption that there exist a negligible loading on the panel by the environment. Under these assumptions one may readily derive

$$g_\infty(x \mid x') = (k_p/2\omega m Y)\exp(-ik_pY \mid x - x' \mid) \quad , \tag{16}$$

$$a_\infty(x \mid x') = \exp(-i\,k_pY \mid x - x' \mid) \quad , \tag{17}$$

and one is warned that Y needs to be picked so that its imaginary part is negative.

[cf.equations (9) and (11).] Without significant loss in generality, k_y is fixed at zero so that Y is rendered unity and out of the way. For the immediate purposes of this paper the line impedances of the ribs are also simply stated. The line impedance of typically the (n)th rib is stated in the form

$$Z_n = i\,\omega\,M_n \quad . \tag{18}$$

When M_n is real and positive, Z_n is a mass controlled line impedance and M_n is then the mass per unit length.

In this paper a specific scheme for displaying the computations is followed. The scheme is devised not only to clarify the meaning of each figure but also to lump, as much as possible, the explanation of the figures. In that vein, (x'/b) is fixed at (-4.7) and only 33 ribs are used. The line impedances of the ribs are assumed mass controlled. The standard values of the parameters used in the computations are: $(M/mb) = 0.3$, $(b\omega_c/c) = 16$, $\eta_p = 5 \times 10^{-3}$, and $(c_\ell/c) = 1.1$; only changes, if any, in these standard parameters will be highlighted. The frequency range covered is $0 < (\omega/\omega_c) < 0.6$. Each of the quantities presented is plotted as a function of $[\,(x - x')\,/b]$ at a discrete normalized frequency (ω/ω_c). With the exception of Figure 6, multitude of plots of a specific quantity are simultaneously presented on a single figure; the plots are those of discrete, successive, and ascending values of the normalized frequency (ω/ω_c). The interferences between such plots are avoided by suppressing the interfering portions of the plots at the higher frequencies.

The form of the magnitude of $a_{\infty}(x \mid x')$; namely,

$$|a_{\infty}(x \mid x')| = |\,g_{\infty}(x \mid x') \,/\, g_{\infty}(x \mid x)\,| = |\exp(-i\,k_p\,|x - x'|)\,| \quad ,$$

is obvious enough whether the surface impedance of the uniform panel is as stated in equation (15a) or (15b), and need not be, therefore, specifically displayed. [cf.equations (9) and (17).] For example, it is obvious that if the loss factor is increased, the drop off in

$|a_{\infty}(x \mid x')|$ with increase in $[\,|x - x'|/b]$ will be correspondingly higher, and so forth. Again, this statement holds whether the surface impedance of the panel is as stated in equations (15a) or (15b). From equations (7) and (17) one may obtain

$$[g(x \mid x')/g_{\infty}(x \mid x)] = a(x \mid x') = a_{\infty}(x \mid x')\,[1 - s(x \mid x')] \quad ;$$
$$s(x \mid x') = \sum_n \sum_r \exp[ik_p(|x - x'| - |x - x_n| - |x_r - x'|)]\, R_n C_{nr} \quad . \tag{19}$$

Since the factor $[1 - s(x \mid x')]$ carries all the description relating to the ribs, and the factor $a_{\infty}(x \mid x')$ is simple enough, it may be convenient and advantageous to deal, in subsequent computational displays, with the former factor only. The representation of $[1 - s(x \mid x')]$ implies that the normalization is specified in terms of a line "force drive". One may, however, specify the normalization in terms of a line "velocity drive". In the latter case, the representation is carried out in terms of $[1 - s(x \mid x')]\,[1 - s(x' \mid x')]^{-1}$. The implication of such a representation lies, however, outside the immediate scope of this paper. Indeed, in this paper, the former representation is rule. In Figure 3a, the magnitude of $[1 - s(x \mid x')]$ is plotted for a periodic R– model, using equation (15a) for the surface impedance of the panel. The phenomena of pass and stop bands are clearly visible. In Figure 3b the magnitude of $[1 - s(x \mid x')]$ is plotted for a periodic R– model, using equation (15b) for the surface impedance of the panel. Figure 3b is similar to Figure 3a and any differences are readily accountable. In particular, the phenomena of pass and stop bands are also clearly visible in Figure 3b. In Figure 3c the conditions imposed on Figure 3b are maintained, except that the loss factor is increased to $\eta_p = 5 \times 10^{-2}$. The subduing of the pass and stop bands by the increase in damping is clearly evidenced by comparing these two figures. In Figures 4a and 4b the magnitude of $[1 - s(x \mid x')]$ is plotted for a periodic NR– model, using equations (15a) and (15b) for the surface impedances of the panel, respectively.[6] The phenomena of pass and stop bands are substantially and clearly missing in these figures. Staying with these gross displays, in Figure 5a the magnitude of $[1 - s(x \mid x')]$ is

plotted for a perioaic R– model, using equation (15b) for the surface impedance of the panel. In this figure, however, the periodic R– model is changed in that the periodic positions of the ribs are "randomly" disturbed by not more than some 10% of the typical separation b between adjacent ribs. The aperture of the array of ribs is kept, however, unchanged. The fading of the pass and stop bands due to this disturbance in the periodicity is clearly evidenced by comparing the figure with its counterpart, Figure 3b.[9] Figure 5b duplicates the conditions of Figure 5a except that the NR– model is substituted for the R– model. In gross terms, the disturbed periodicity hardly effects the NR– model as can be deduced by comparing Figure 5b with Figure 4b.[9] One may already deduce from the presented figures, the associations that exist between the fading of the pass and stop bands and the manifestation of localizations and delocalizations. It may, however, be useful to single out some of these features so that one can see the trees in the forest. For this purpose Figure 6 is offered. In Figure 6, two groups of three individual plots of the magnitude of $[1 - s(x \mid x')]$ are depicted. In Figure 6a, one group is representing the periodic R– model and the other, the periodic NR– model. In Figure 6b, one group is representing the random R– model and the other, the periodic NR– model. In Figure 6c, one group is representing the random R– model and the other, the periodic R– model. The individual plots are for three distinct normalized frequencies; the top at (ω_s/ω_c), the middle at (ω_p/ω_c),, and the bottom at (ω_{ps}/ω_c), where ω_s lies in a stop band, ω_p lies in an adjacent pass band, and ω_{ps} lies in between the pass band and the subsequent stop band. [cf. Figures 3b and 4b.] Figure 6b duplicates the conditions of Figure 6a except that the periodicity of the R– model is "randomly" disturbed by not more than some 10% of the nominal separation b. [cf. Figures 5a and 4b.] The occurrence of the conventional localizations and delocalizations is clearly exhibited in Figure 6c.

It is deemed that much of what was stated and argued in the preceding section has withstood experimental verifications; the experiments being conducted on a computer. There is much to be investigated in this and related areas of structural acoustics so that

some of the notions and results expressed and derived herein may be put to practical use. A little of this kind of use is immediately obvious; e.g., one should be careful of the cavalier use of randomizing the spacing between the ribs to achieve localizations, hoping to reduce the transmission in an intended frequency band. One may simultaneously cause delocalization with accompanied increase in transmission at another frequency band which may, even on balance, be detrimental; not to mention that were other modes of propagation present, the expected benefit may not materialize even in the intended frequency band. More will develop with further research.[4,9]

Remarks

1. The advantage lies in the manipulation and interpretation of the equation. However, if the final quantity is required to be in real space a price of inverse transformation must be paid.

2. The examples and computations provided and carried out in this paper are chiefly based on material derived and discussed in Reference 1.

3. As Figure 2 explains, it is essential, in constructing the NR– model, to retain the distinction between $a_\infty(x_j \mid x_i)$ and $a_\infty(x_i \mid x_j)$, or equivalently, retain in $a_\infty(x_j \mid x_i)$ the distinction between $x_i < x_j$ and $x_i > x_j$. Since for a uniform panel, $a_\infty(x \mid x')$ is stationary and reciprocal in x; i.e., $a_\infty(x \mid x') = a_\infty(x - x') = a_\infty(x' - x)$, the required distinction is often lost early in the development of many formalisms. However, the formalism developed in Reference 1 does retain this distinction. The retention is crucial in the construction of the NR– model. Moreover, the removal of the return interactions is most conveniently performed on the primitive matrix $\underset{\approx}{B}$, rather than on the compounded matrix $\underset{\approx}{T}$. In this connection, one should not confuse the NR– model with the (N)th order model. In the former modeling scheme, certain specific type of interactions; namely, the return interactions, are omitted. In the latter modeling scheme: in the zeroth order model, the nonuniformities are absent altogether; in the first order model, a nonuniformity is oblivious to all other nonuniformities; in the second order model, a pair of adjacent nonuniformities is oblivious to all other nonuniformities; and so on [4,5].

4. If additional modes of propagation and environmental loadings are incorporated, the analysis becomes more and more compounded and requires more study. Since many practical dynamic systems simultaneously support several modes of propagation and they are usually loaded by the environment, this is indeed an area to be explored.

5. A regular model is one in which the nonuniformities are identical <u>and</u> equally spaced. One may introduce deviations in the identities of the nonuniformities and/or in the equalities

of the spacings; if these deviations are randomly selected, the model may be designated a random model in the various deviations [3]. Models that are similarly randomized may form an ensemble. Then a model that is designated a statistical model may be defined by establishing various ensemble averagings [3]. In this paper for the most part the monuniformities (the ribs) are assumed to be identical and, therefore, the spacings of the ribs are central to the modeling. To emphasize this adaptation a model is designated "periodic" when the ribs are equally spaced and is designated "random" when the equality of the spacings are mildly, but randomly, disturbed. It is further noted in this connection that the identities of the ribs -- the equalities of the line impedances of the ribs -- usually are less significant in causing reinforcements or cancellations in the response of the panel. Mild deviations and/or variations in the equalities of the line impedances of the ribs usually do not effect significantly the phases in the return interactions. However, one can introduce line impedances that are so design as to cause pronounced influence on these phases. In this paper, however, such unusual designs are not pursued. It is merely pointed out, in passing, that were the separations between adjacent ribs maintained equal, the introduction of normal inequalities in the line impedances of the ribs, will not subdue the phenomenon of wavenumber aliasing [7].

6. When the phases pertaining to a periodic R– model cannot be made to cause either substantial reinforcements or substantial cancellations; i.e., at frequencies that lie between adjacent pass and stop bands, one may expect that at these frequencies the impulse response functions of a periodic R– model and a corresponding periodic NR– model will exhibit similar behavior.

7. There are numerous statistical techniques and methodologies that can be used to derive the statistical impulse response functions of dynamic systems that incorporate various nonuniformities. There is no attempt in this paper to employ one or another of these techniques and methodologies, and to define specific distributions for the deviations and/or variations that one may attribute to the parameters that specify the dynamic system and/or

its nonuniformities. At best, when a statistical example is called for, a simple member of a simple ensemble is depicted rather than a typical member [3]. This procedure is sufficient to satisfy the purpose and the scope assigned to this paper.

8. It is speculated that in parallel to Anderson localizations, there are to be found, in physical systems that harbor periodicity, the phenomenon of delocalizations that are exhibited when the periodicity is disturbed. In the physical system of ribbed panels here considered, the phenomenon of delocalizations is discovered.

9. The appearances, in Figures 5 and 6, of some "remote" localizations and delocalizations is a matter to be covered in a subsequent paper. Remote is a designation reserved to describe a localization (or a delocalization) that occurs away from the location of the drive. Moreover, the locations of such remote localizations and delocalizations may be manipulated by assigning specific distributions to the positions and the line impedances of the ribs. The apriori choice of such locations is, in principle, a form of passive control of remote localized responses. Again, the cavalier use of this procedure need be carefully examined in a practical situation in which a number of modes of propagation and environmental loadings may be involved.

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Figure Captions

Figure 1. A sketch of a ribbed panel showing the coordinate system and the orientations and locations of the ribs x_j's, the drive x', and the observation x.

Figure 2. Examples of nonreturn and corresponding return interactions between a pair of ribs.

a. A nonreturn interaction: $B^0_{ji} = B_{ji}$; $x_i < x_j$.

b. A return interaction: $B^0_{ij} = 0$; $x_i < x_j$.

c. A return interaction: $B^0_{ji} = 0$; $x_i > x_j$.

Figure 3. A composite of the magnitude of the normalized impulse response function $[1 - s\,(x \mid x')]$ of a periodic R– model of a ribbed panel, as a function of the normalized distance $[(x-x')\,/b]$ from the position x' of the line drive, for discrete, successive, and ascending values of the normalized frequency (ω/ω_c).

a. A membrane-like panel; equation (15a).

b. A membrane simulating a plate responding in flexure; equation (15b).

c. As in Figure 3b except that damping is increased from the standard value of $\eta_p = 5 \times 10^{-3}$ to $\eta_p = 5 \times 10^{-2}$.

Figure 4a. As in Figure 3a except that the periodic NR– model is substituted for the periodic R– model .

Figure 4b. As in Figure 3b except that the periodic NR– model is substituted for the standard R– model.

Figure 5. A composite of the magnitude of the normalized impulse response function $[1 - s\,(x \mid x')]$ of a ribbed panel described by equation (15b) as a function of the normalized distance $[\,|x-x'|\,/b]$ from the position x' of the line drive, for discrete, successive, and ascending values of the normalized frequency (ω/ω_c). The separations between adjacent ribs are disturbed from equality by, at most, some 10% of the nominal separation b.

a. R– model. [cf. Figure 3b.]

b. NR– model. [cf. Figure 4b.]

Figure 6. A composite of the magnitude of the normalized impulse response function $[1 - s\,(x \mid x')]$ of a ribbed panel described by equation (15b) as a function of the normalized distance $[(x-x')\,/b]$ from the position x' of the line drive, for three discrete and descending values of the normalized frequency (ω/ω_c): (ω_s/ω_c), (ω_p/ω_c), and (ω_{ps}/ω_c), where (ω_s/ω_c) lies in a stop band, (ω_p/ω_c) lies in the adjacent pass band, and (ω_{ps}/ω_c) lies between the pass band and the subsequent stop band.

a. Comparison between a periodic R– model (darker) and a periodic NR– model (lighter).

b. Comparison between a random R– model (darker) and a periodic NR– model (lighter). A recovered localization and a recovered delocalization are exhibited.

c. Comparison between a random R– model (darker) and a periodic R– model (lighter). A conventional localization and a conventional delocalization are exhibited.

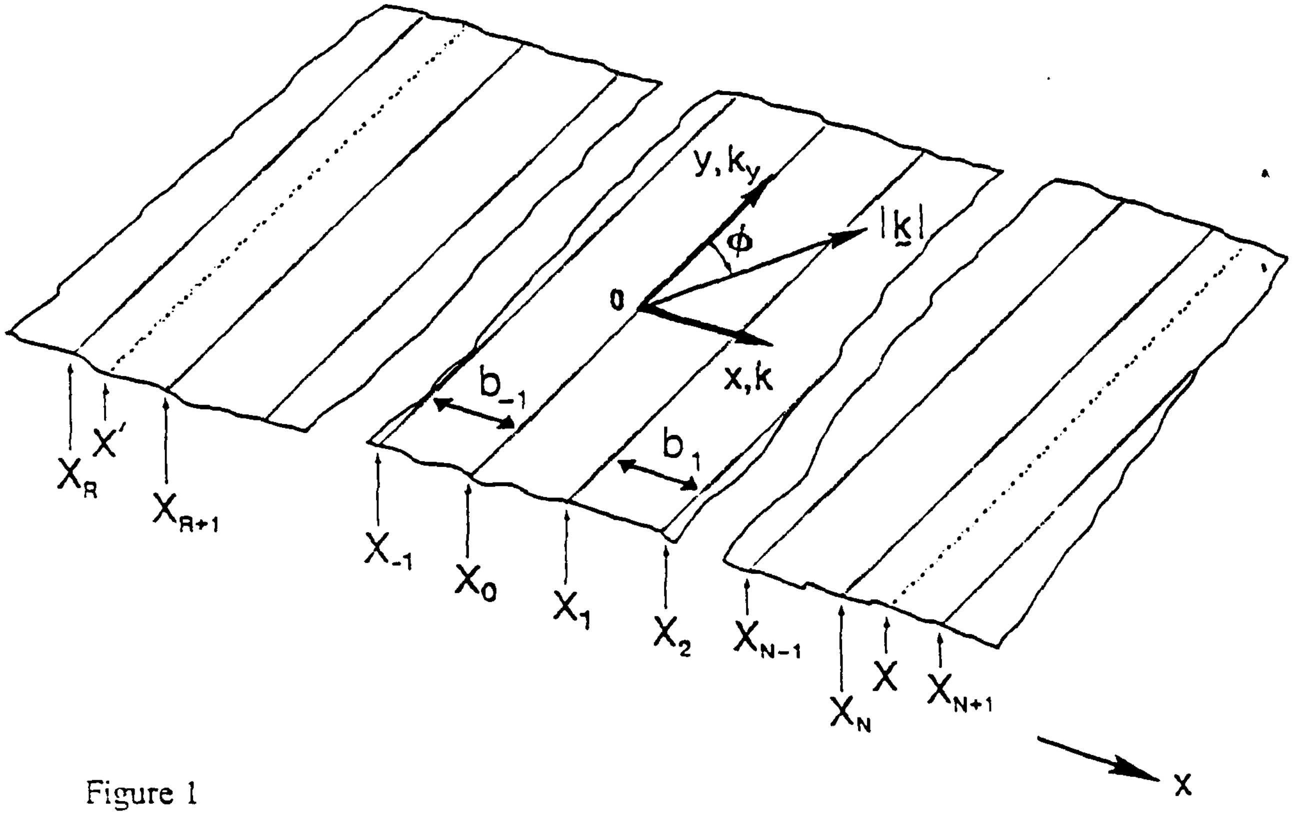

Figure 1

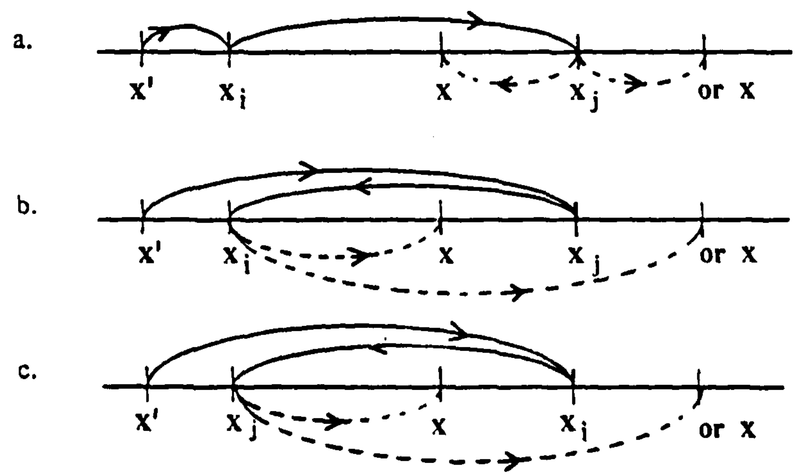

Figure 2

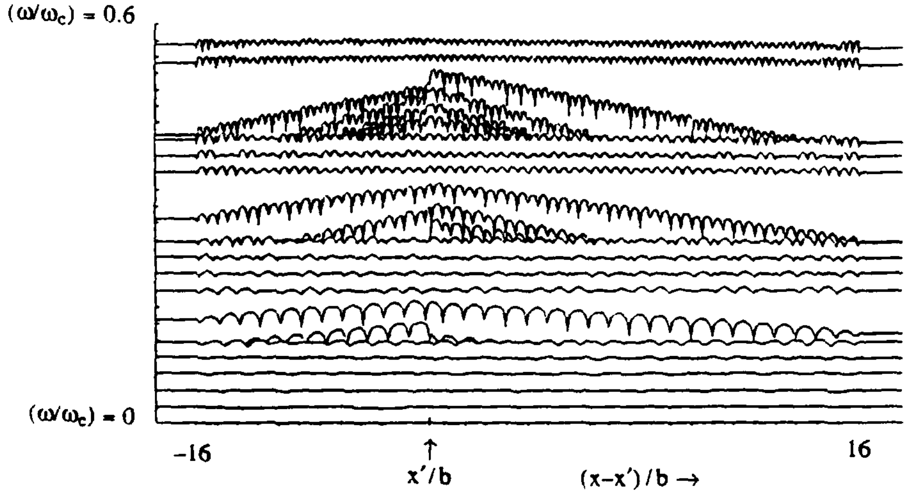

Figure 3a

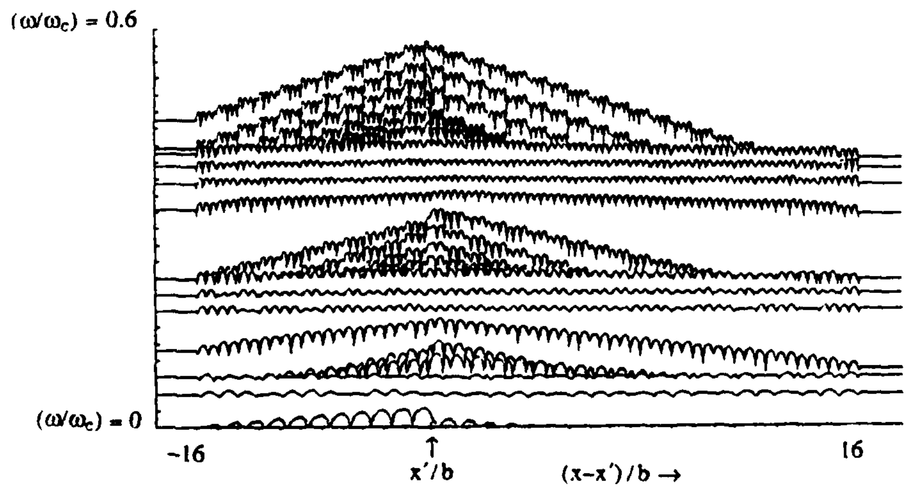

Figure 3b

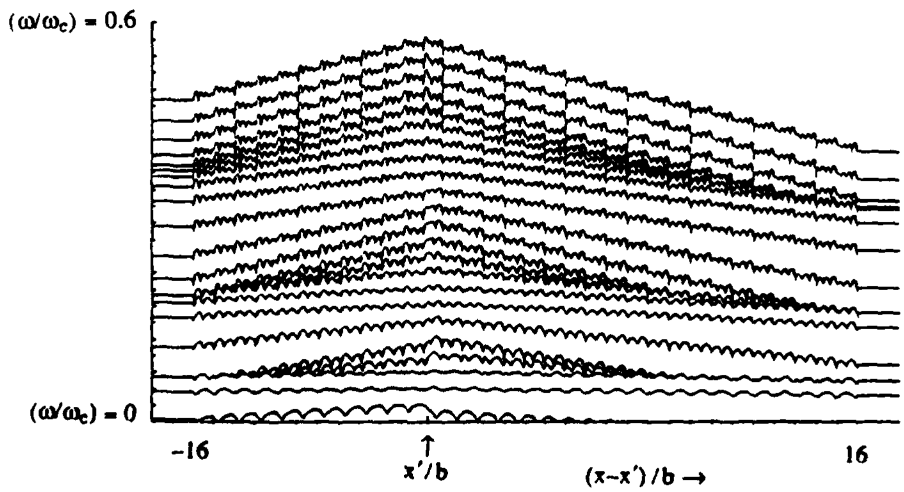

Figure 3c

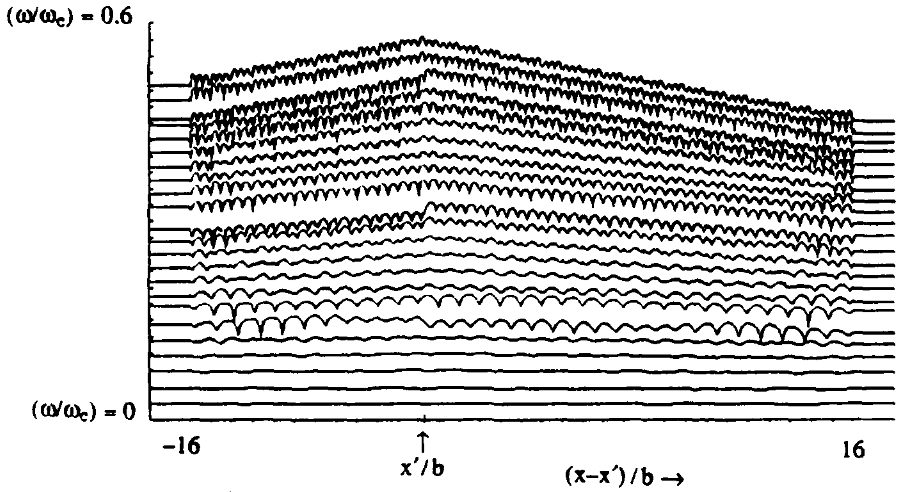

Figure 4a

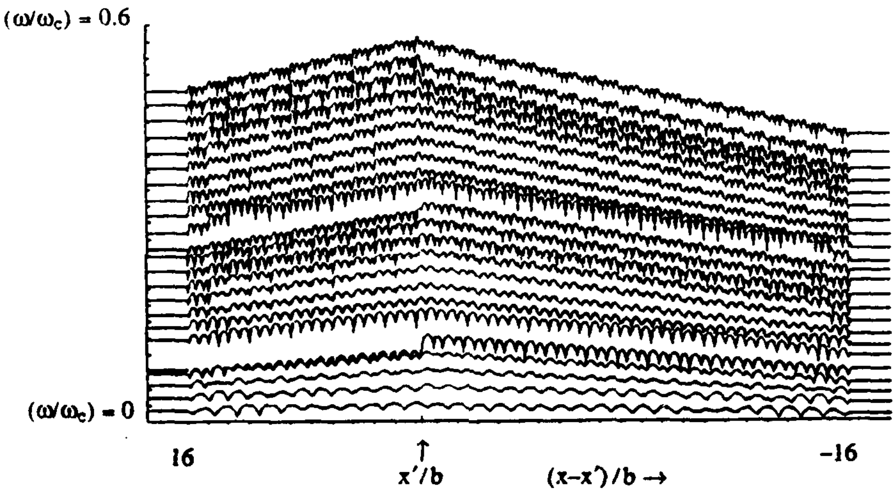

Figure 4b

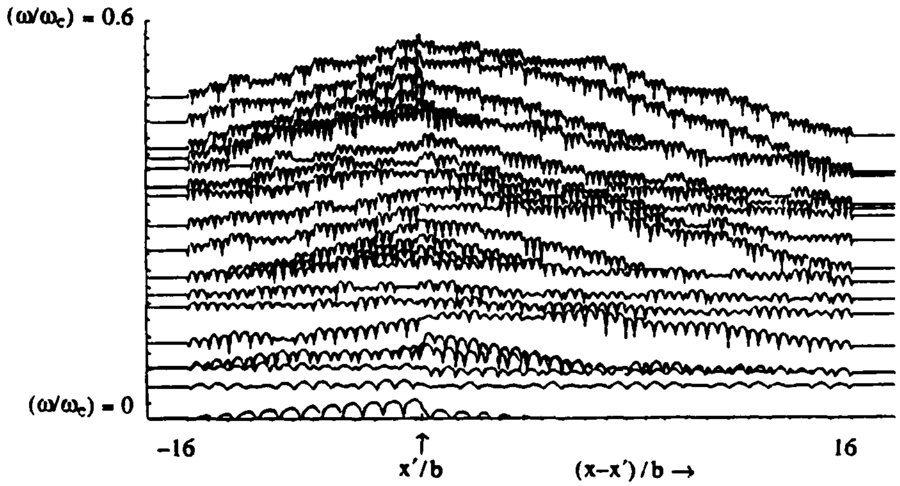

Figure 5a

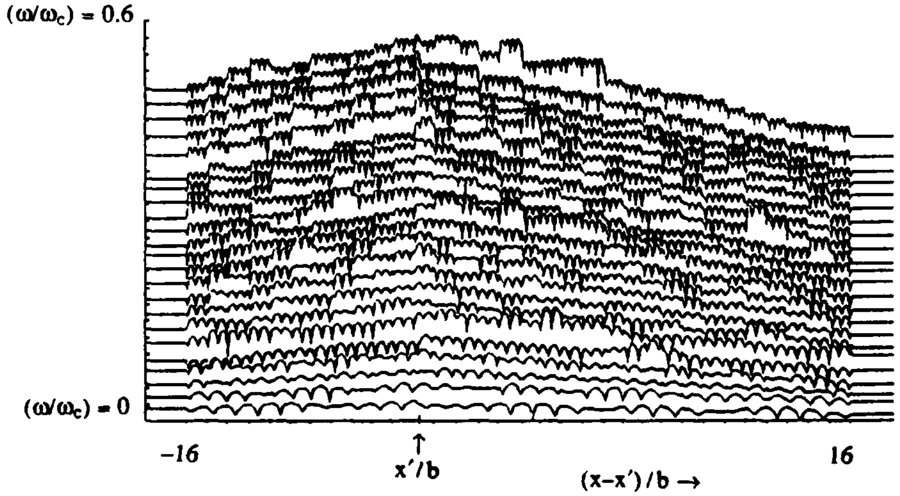

Figure 5b

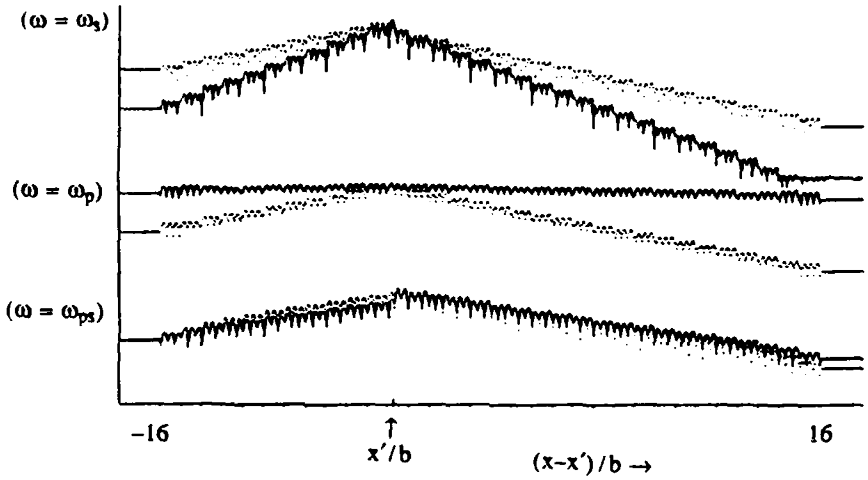

Figure 6a

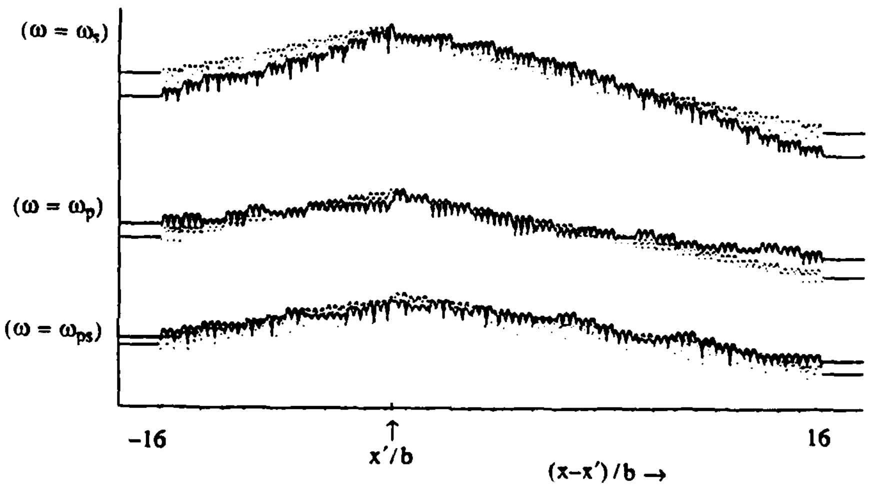

Figure 6b

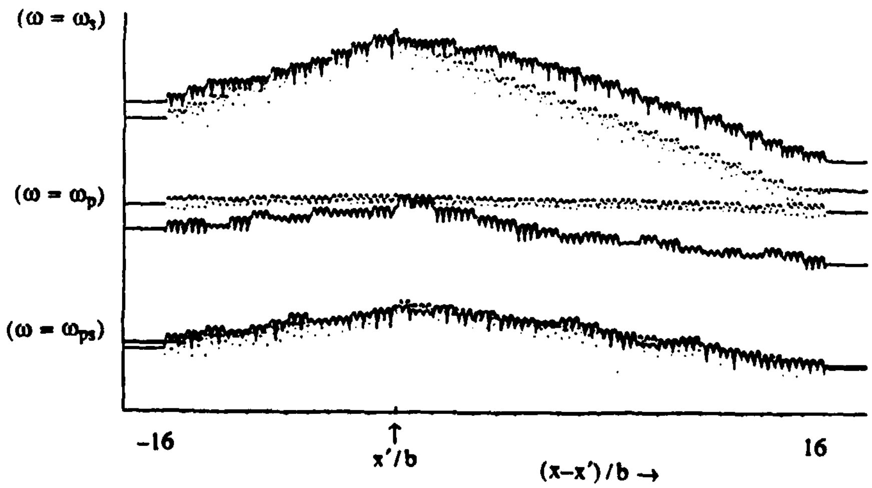

Figure 6c

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